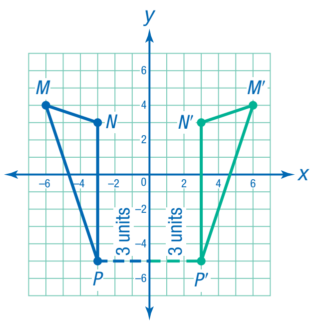
LESSON 3 Reflections

UNDERSTAND A **reflection** is a transformation that flips a figure across a line called a **line of reflection**. Each reflected point is the same distance from the line of reflection as its corresponding point on the preimage, but it is on the opposite side of the line. The resulting image and the preimage are mirror images of one another. The line of reflection can be the *x*-axis, the *y*-axis, or any other line in the coordinate plane.

You can think of a reflection of a figure as a function in which the input is not a single value, *x*, but rather a point on the coordinate plane, (x, y). When you apply the function to a point on a figure, the output will be the coordinates of the reflected image of that point.

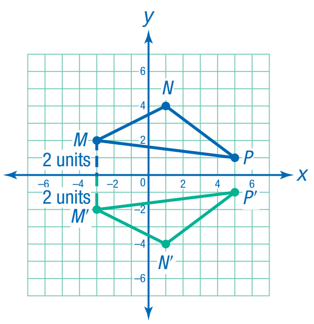
When a point is reflected across the *y*-axis, the sign of its *x*-coordinate changes. The function for a reflection across the *y*-axis is:

$$R_{y\text{-axis}}(x, y) = (-x, y)$$



When a point is reflected across the *x*-axis, the sign of its *y*-coordinate changes. The function for a reflection across the *x*-axis is:

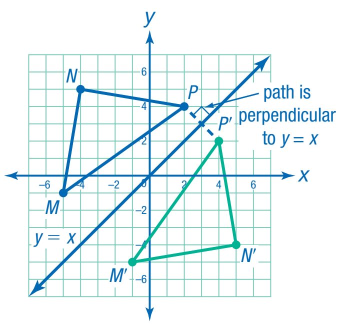
$$R_{x\text{-axis}}(x, y) = (x, -y)$$



Another common line of reflection is the diagonal line $y = x$. To reflect over this line, swap the *x*- and *y*-coordinates. The function for a reflection across line $y = x$ is:

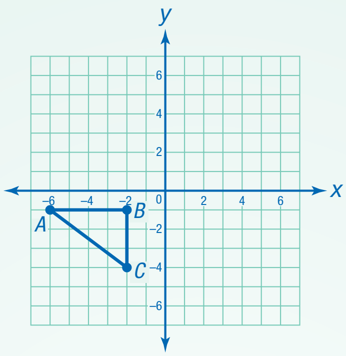
$$R_{y = x}(x, y) = (y, x)$$

The path that a point takes across the line of reflection is always **perpendicular** to the line of reflection. Perpendicular lines form right angles when they cross one another. As shown in the diagram on the right, the path from point P to point P' forms right angles with the line of reflection, $y = x$.



Connect

Reflect $\triangle ABC$ across the x-axis. Then reflect $\triangle ABC$ across the y-axis.



1 Identify the coordinates of the vertices of $\triangle ABC$.

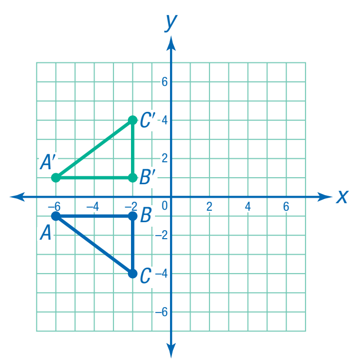
The vertices of the triangle are $A(-6, -1)$, $B(-2, -1)$, and $C(-2, -4)$.

2 To reflect the vertices of $\triangle ABC$ across the x-axis, change the signs of the y-coordinates. Then draw the image.

$A(-6, -1) \rightarrow A'(-6, 1)$

$B(-2, -1) \rightarrow B'(-2, 1)$

$C(-2, -4) \rightarrow C'(-2, 4)$

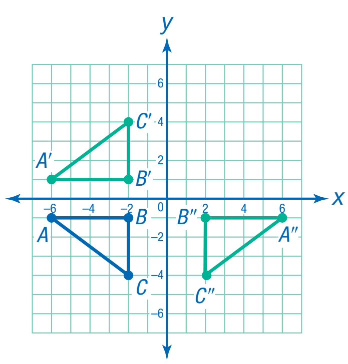


3 To reflect the vertices of $\triangle ABC$ across the y-axis, change the signs of the x-coordinates. Then draw the image.

$A(-6, -1) \rightarrow A''(6, -1)$

$B(-2, -1) \rightarrow B''(2, -1)$

$C(-2, -4) \rightarrow C''(2, -4)$



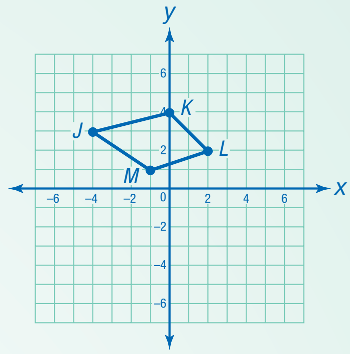
TRY

Use function notation to describe how $\triangle ABC$ is transformed to $\triangle A'B'C'$ and how $\triangle ABC$ is transformed to $\triangle A''B''C''$.

EXAMPLE A Graph the image of quadrilateral *JKLM* after the reflection described below.

$F(x, y) = (y, x)$.

Then describe the reflection in words.



1 Identify the coordinates of the vertices of quadrilateral *JKLM*.

The quadrilateral has vertices $J(-4, 3)$, $K(0, 4)$, $L(2, 2)$, and $M(-1, 1)$.

2 Apply the function to the vertices.

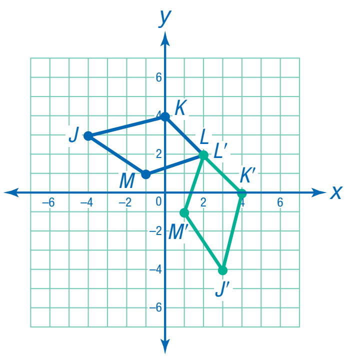
$F(-4, 3) = (3, -4)$

$F(0, 4) = (4, 0)$

$F(2, 2) = (2, 2)$

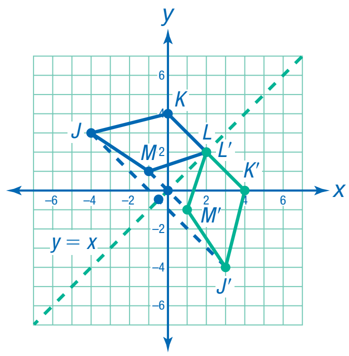
$F(-1, 1) = (1, -1)$

3 Graph the image.



4 Describe the reflection in words.

Find the line that lies halfway between corresponding points of the figure.



Each of these halfway marks lies on the line $y = x$.

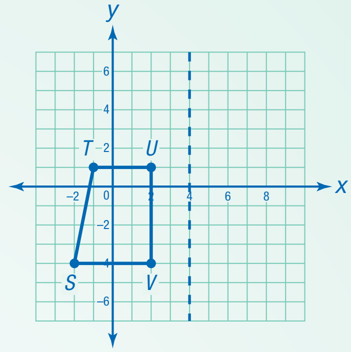
▶ The function performs a reflection across the line $y = x$.

TRY

Apply the same function, $F(x, y) = (y, x)$, to $J'K'L'M'$. What image results?

Figures can be reflected over horizontal or vertical lines that are not the x- or y-axis as well.

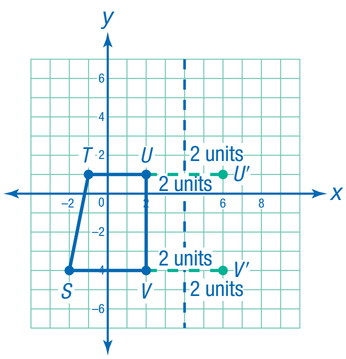
EXAMPLE B Trapezoid *STUV* is graphed on the right. Reflect this trapezoid over the line $x = 4$.



1 Reflect vertices *U* and *V*.

Point *U*, at (2, 1), is 2 units to the left of $x = 4$. So, its reflection will be 2 units to the right of $x = 4$. So, plot a point at (6, 1) and name it U'.

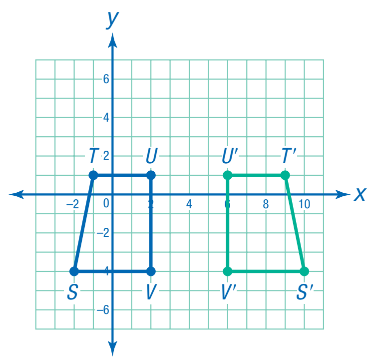
Use the same strategy to plot point V'.



2 Find and plot the other two points of the image.

Point *T* at (−1, 1) is 5 units to the left of $x = 4$. So, plot point T' 5 units to the right of $x = 4$ at (9, 1).

Point *S* is 6 units to the left of $x = 4$. So, plot point S' at (10, −4), which is 6 units to the right of $x = 4$.



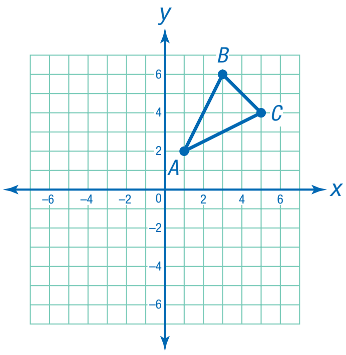
DISCUSS

How could you describe the reflection of trapezoid *STUV* over the line $x = 4$ using function notation?

Practice

Draw each reflected image as described and name its vertices. Identify the coordinates of the vertices of the image.

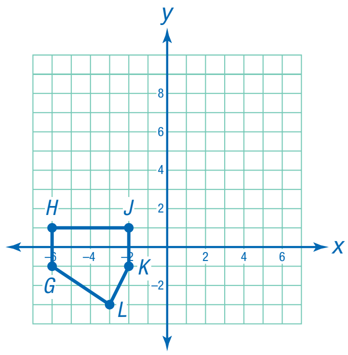
1. Reflect $\triangle ABC$ across the x-axis.



$A'(\_\_\_, \_\_\_)$ $B'(\_\_\_, \_\_\_)$ $C'(\_\_\_, \_\_\_)$

REMEMBER When a point is reflected across the x-axis, the sign of its y-coordinate changes.

2. Reflect pentagon $GHJKL$ across the line $y = 3$.



$G'(\_\_\_, \_\_\_)$ $H'(\_\_\_, \_\_\_)$ $J'(\_\_\_, \_\_\_)$

$K'(\_\_\_, \_\_\_)$ $L'(\_\_\_, \_\_\_)$

Fill in each blank with an appropriate word or phrase.

3. A reflection results in two figures that look like ____________________ of each other.

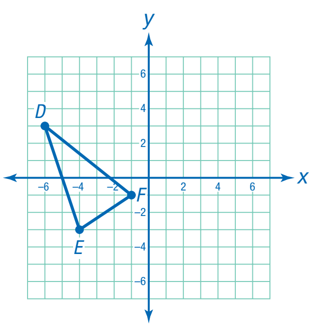
4. Lines that meet and form right angles are called _______________ lines.

5. A point and its reflection are each the same distance from _________________________.

6. The path that a point takes across the line of reflection is ____________ to the line of reflection.

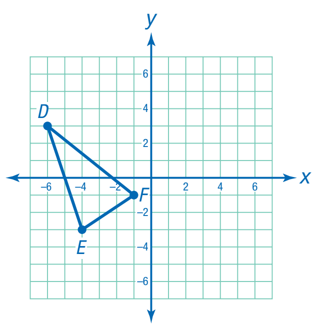
Use the given function to transform $\triangle DEF$. Then describe the transformation in words.

7. $R(x, y) = (-x, y)$



____

8. $R(x, y) = (y, x)$



____

Identify the coordinates of the image for each reflection as described.

9. Reflect $M(3, 4)$ across the x-axis.

$M'(\_\_\_\_\_, \_\_\_\_\_)$

10. Reflect $N(-2, -8)$ across the y-axis.

$N'(\_\_\_\_\_, \_\_\_\_\_)$

11. Reflect $P(-2, 0)$ across the line $y = x$.

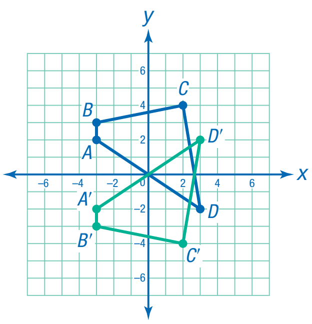
$P'(\_\_\_\_\_, \_\_\_\_\_)$

12. Reflect $Q(5, 10)$ across the line $y = x$.

$Q'(\_\_\_\_\_, \_\_\_\_\_)$

Describe how quadrilateral *ABCD* was reflected to form quadrilateral *A′B′C′D′*, using both words and function notation.

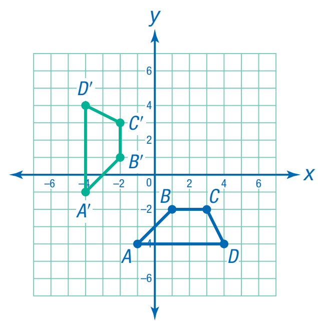
13.



Words: ____________________

Function: ____________________

14.

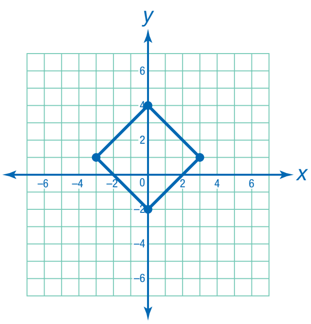


Words: ____________________

Function: ____________________

Solve.

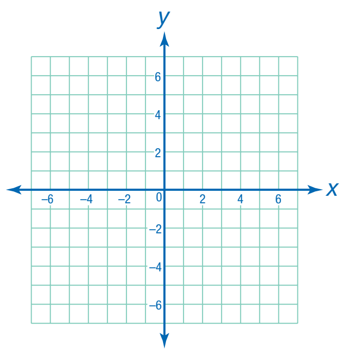
15. **JUSTIFY** Camille drew the square below on a coordinate plane. She says that if she reflects the square over the x-axis it will look exactly the same as if she reflects it over the y-axis. Is she correct or incorrect? Use words, numbers and/or drawings to justify your answer.



____

____

16. **DRAW** Patrick reflected a figure in two steps. The result was that each point (x, y) was transformed to the point $(-y, x)$. Draw a triangle (any triangle) on the plane below and transform it as described. Then describe what two reflections Patrick performed.



____

____